STOCHASTIC USER EQUILIBRIUM WITH EQUILIBRATED CHOICE SETS

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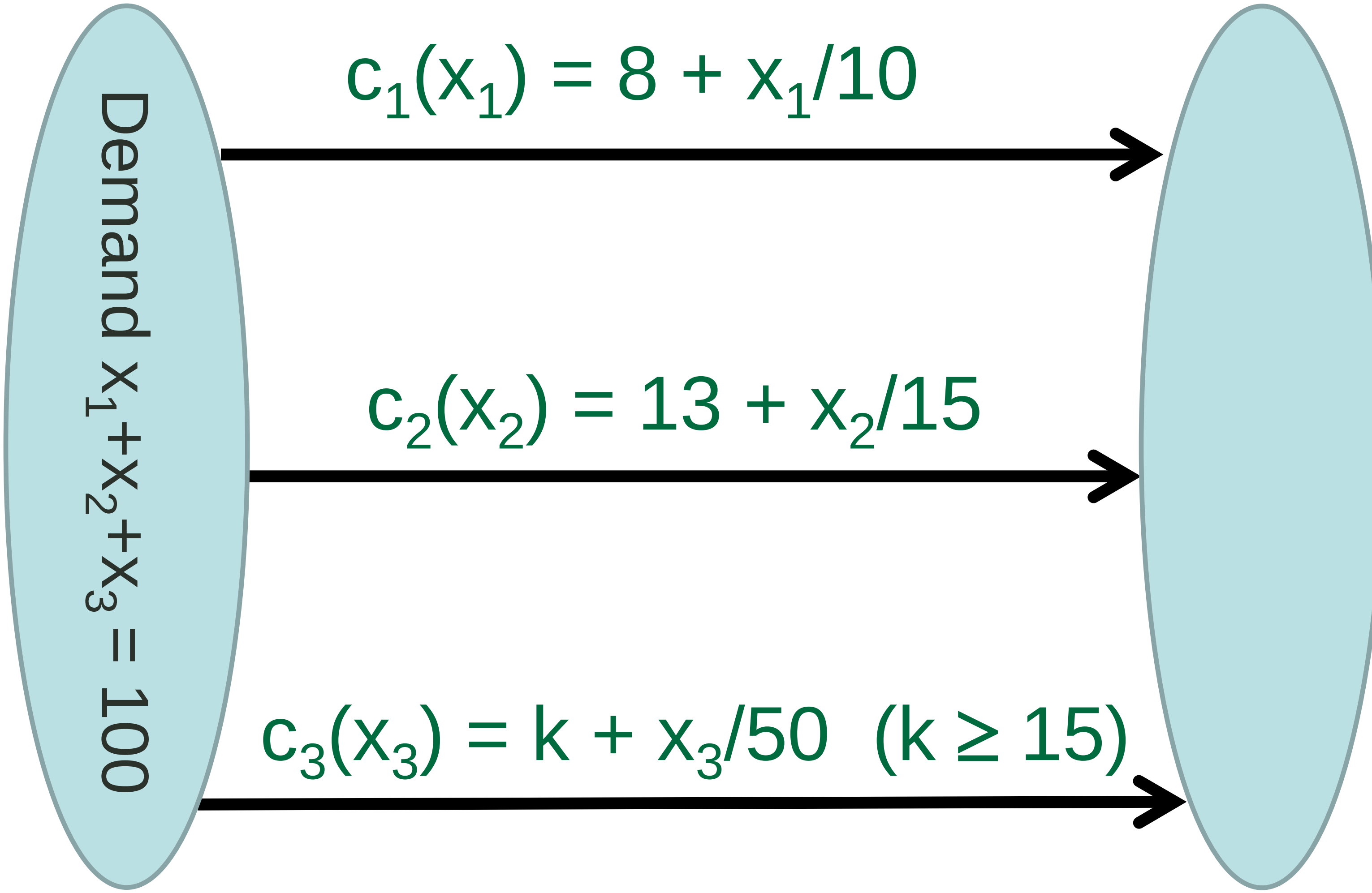
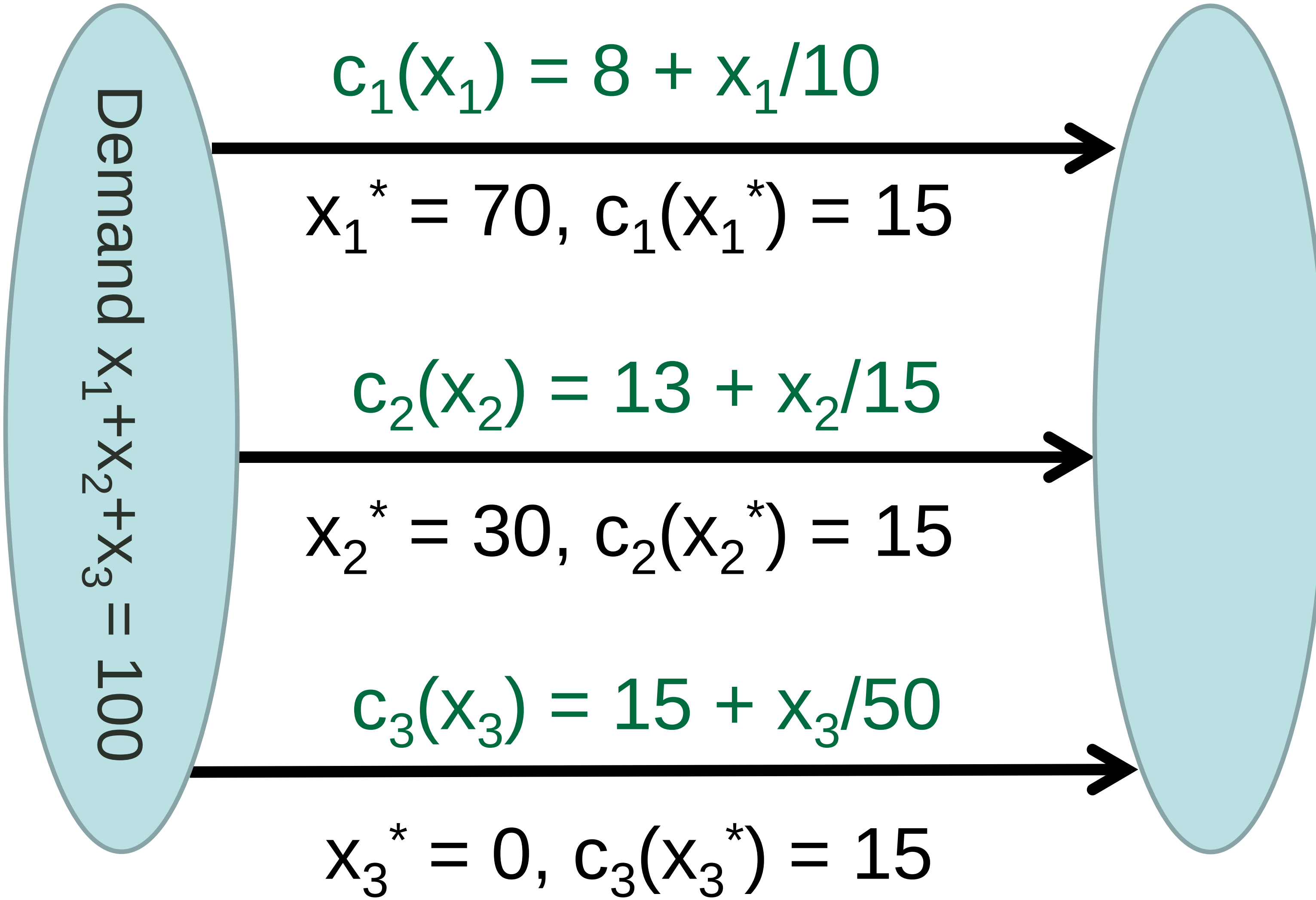
Example



Deterministic User Equilibrium (DUE)
"Used routes have equal, minimal cost"



Deterministic User Equilibrium (DUE)
"Used routes have equal, minimal cost"

Demand $x_1+x_2+x_3 = 100$

$c_1(x_1) = 8 + x_1/10$

$x_1^* = 70,\ c_1(x_1^*) = 15$

$c_2(x_2) = 13 + x_2/15$

$x_2^* = 30,\ c_2(x_2^*) = 15$

$c_3(x_3) = 20 + x_3/50$

$x_3^* = 0,\ c_3(x_3^*) = 20$

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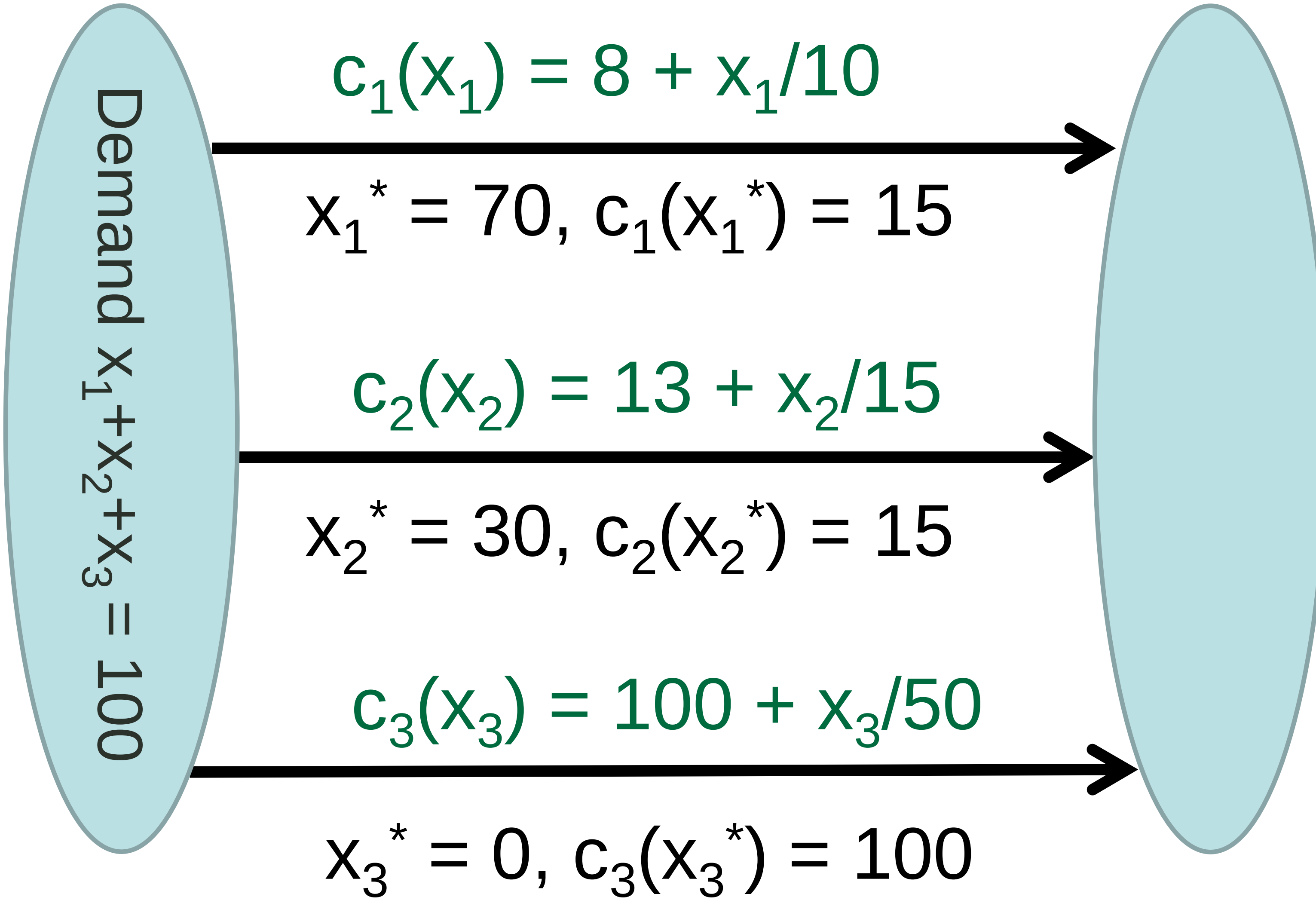
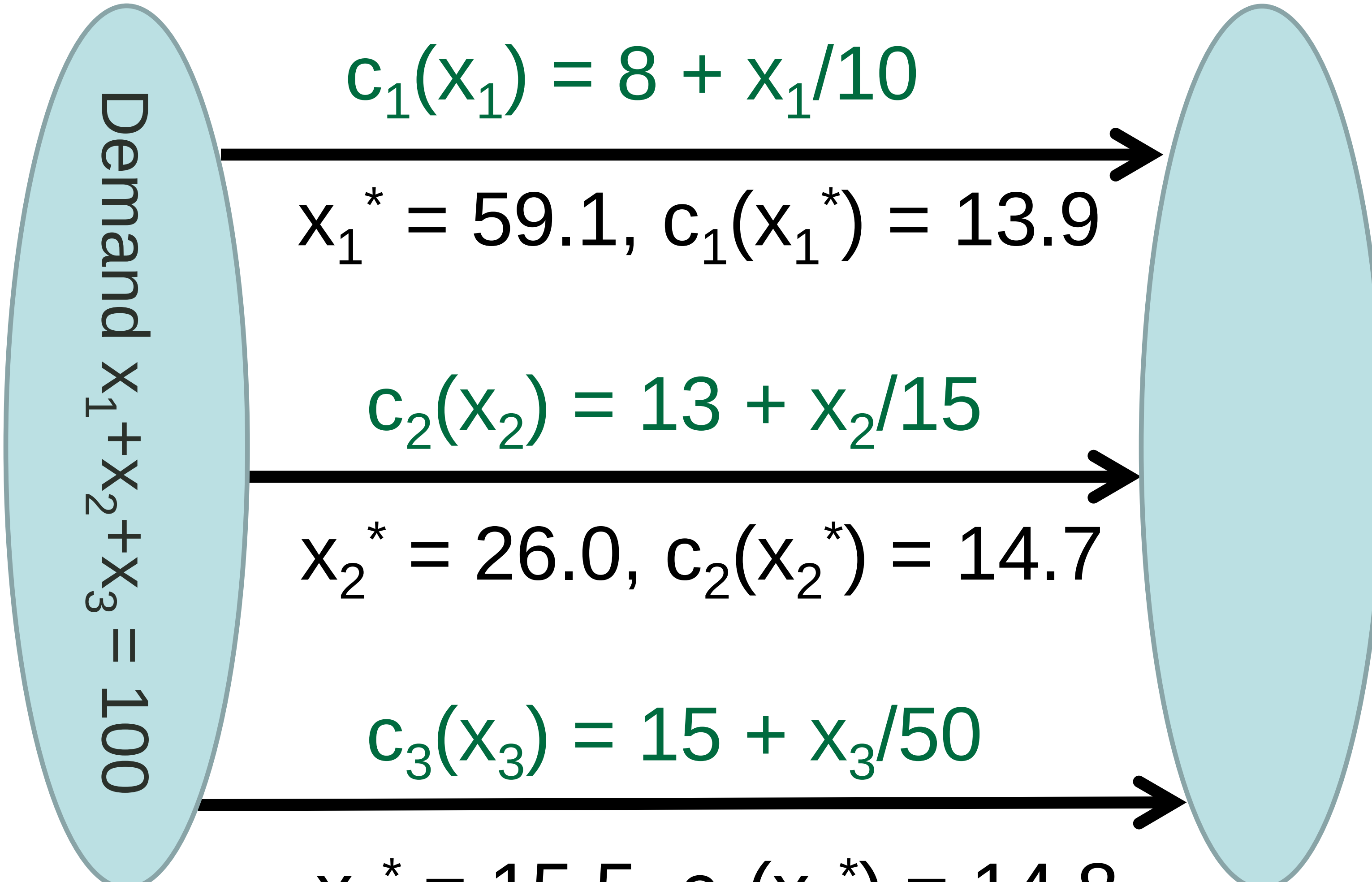
Deterministic User Equilibrium (DUE)
“Used routes have equal, minimal cost”

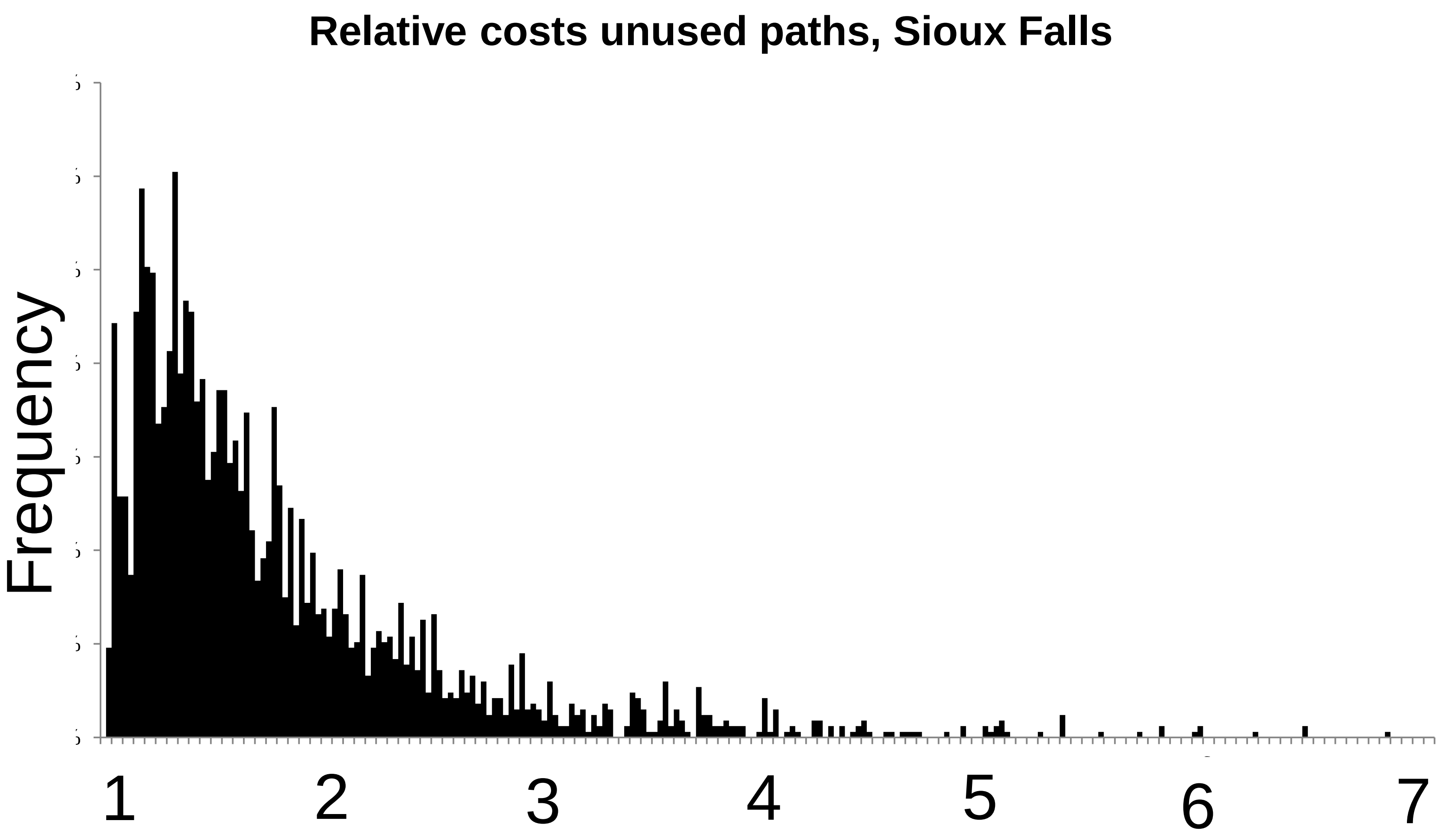


Stochastic User Equilibrium (SUE)
"Logit: x_r proportional to $\exp(-\theta c_r)$"



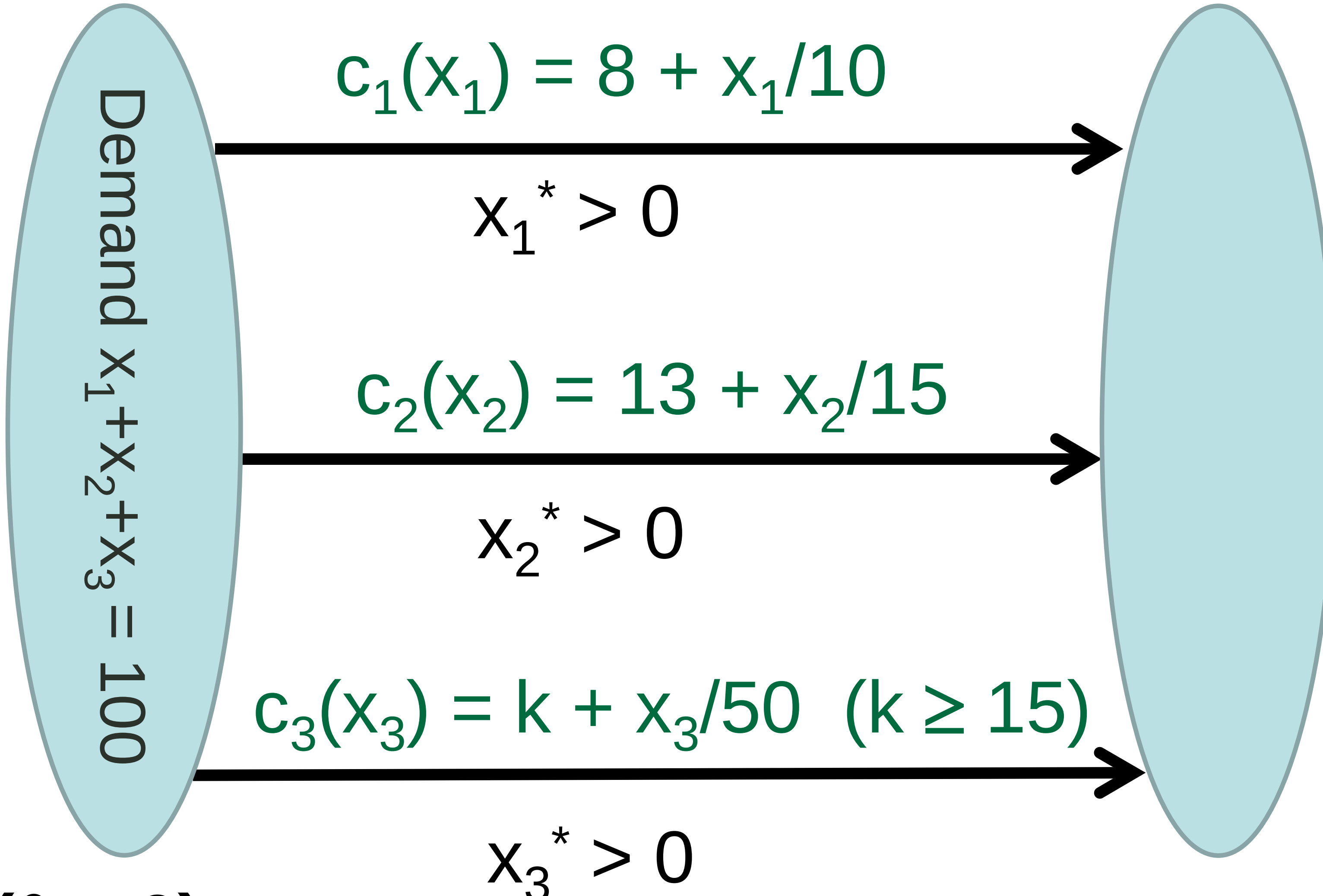
($\theta = 1$)

DUE model in a larger network: unused paths



Horizontal axis = Cost of an unused path / Cost of any used path for all ODs

Stochastic User Equilibrium (SUE)
"Logit: x_r proportional to $\exp(-\theta c_r)$"



$(\theta > 0)$

Objectives

- To explore alternative definitions of equilibrium mixing random utility with used/unused routes.
- Aim for a self-consistent model.
- Only use same data as typically available for large scale network analysis.
- Practical solution algorithms and validation considered later (see following presentation).

Deterministic User Equilibrium (DUE)

1. All used routes have the same travel cost.

2. Reference cost = cost of any used path

3. Unused path cost ≥ reference cost.

(for each OD movement)

Stochastic User Equilibrium (SUE):

1. Routes are used in proportion[1] to $\exp(-\theta c_r)$

(for all possible routes[2] for each OD movement)

[1]For the simplest case of logit model, or in other proportions for other discrete choice models.

[2]The number of possible routes can be enormous for large realistic networks.

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Restricted SUE model (RSUE(min))

1. Used routes used in proportion to $\exp(-\theta c_r)$.

2. Reference cost = min {cost of any used path}

3. Unused path cost $\geq$ reference cost.

(for each OD movement)

Restricted SUE model (RSUE(max))

1. Used routes used in proportion to $\exp(-\theta c_r)$.

2. Reference cost = max {cost of any used path}

3. Unused path cost $\geq$ reference cost.

(for each OD movement)

General Properties

Any SUE solution is also a RSUE(min) and RSUE(max) solution. The converse is not true (see example). So RSUE widens the set of possibilities relative to SUE (i.e. it is a relaxation).

Any RSUE(max) solution is also a RSUE(min). The converse is not true (see example).

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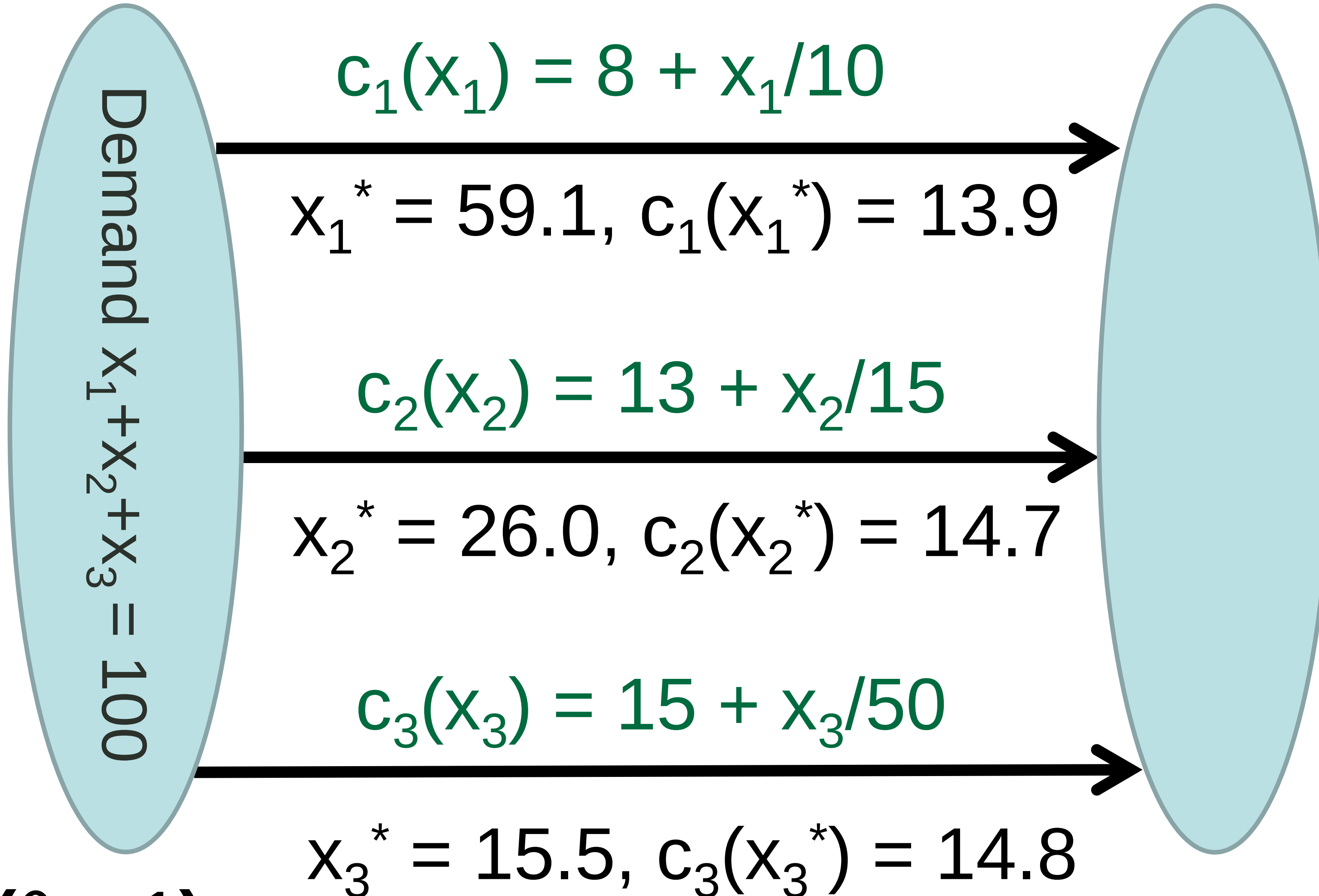
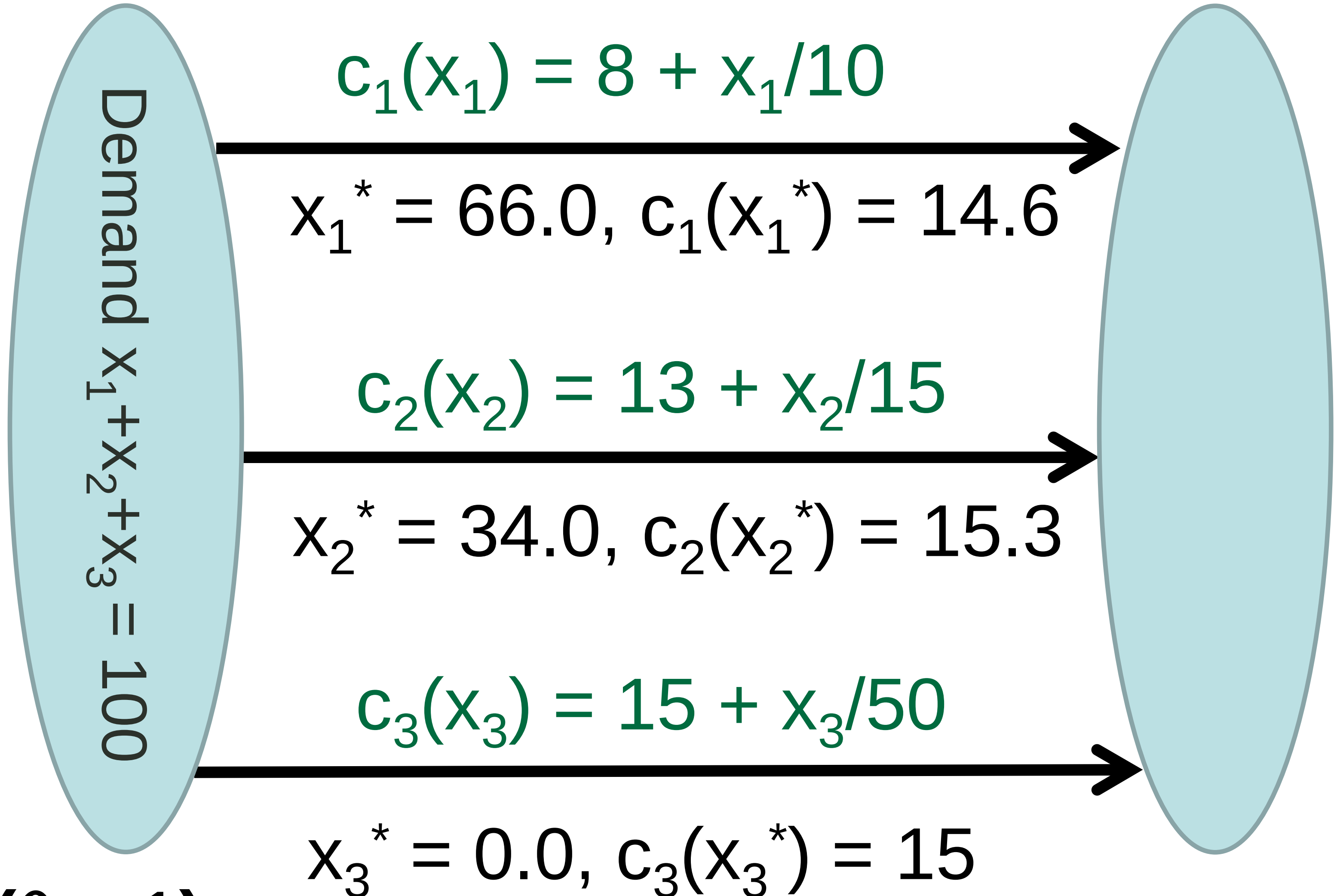
RSUE(min) solution 1 (= SUE)



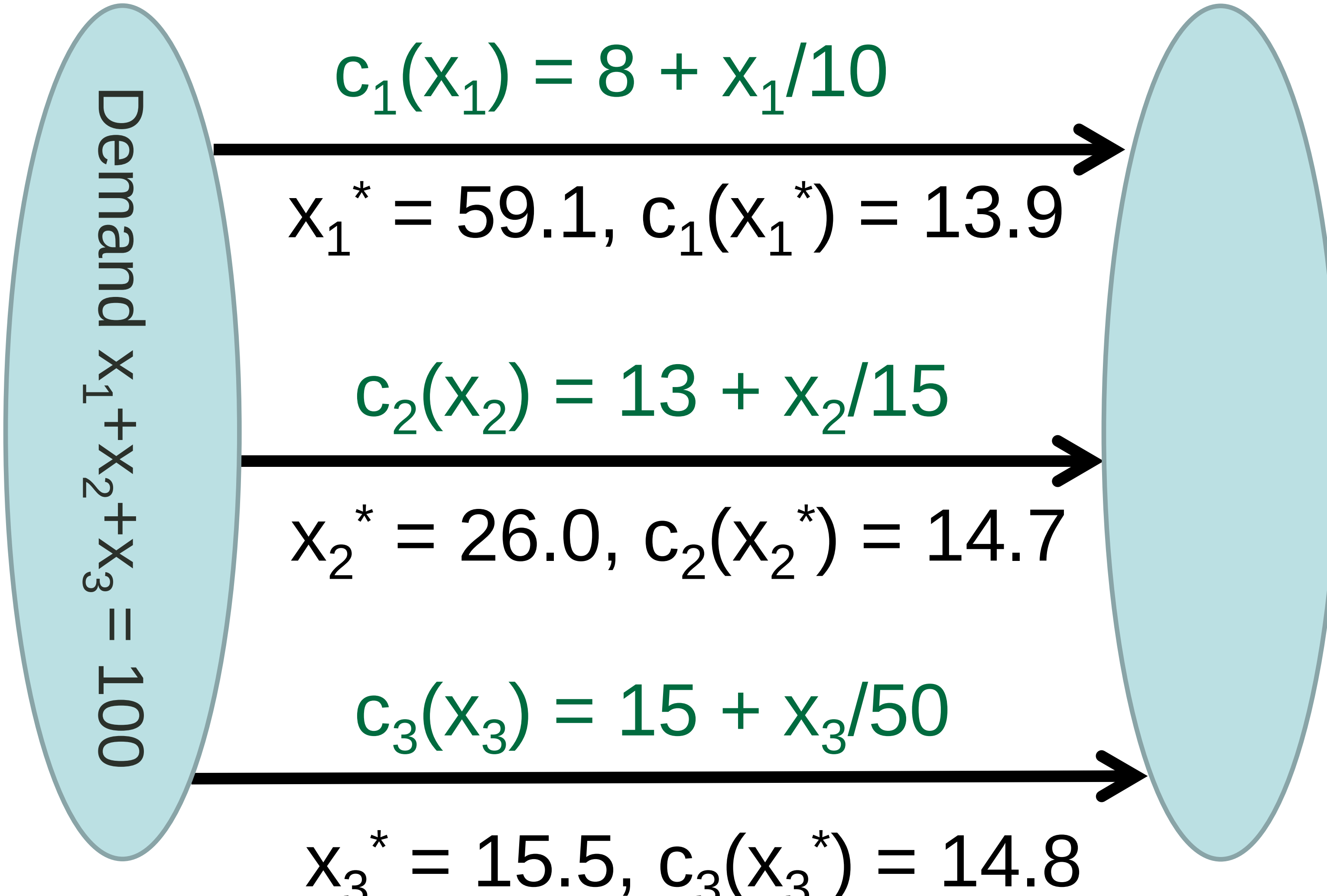
($\theta = 1$)

RSUE(min) solution 2



($\theta = 1$)

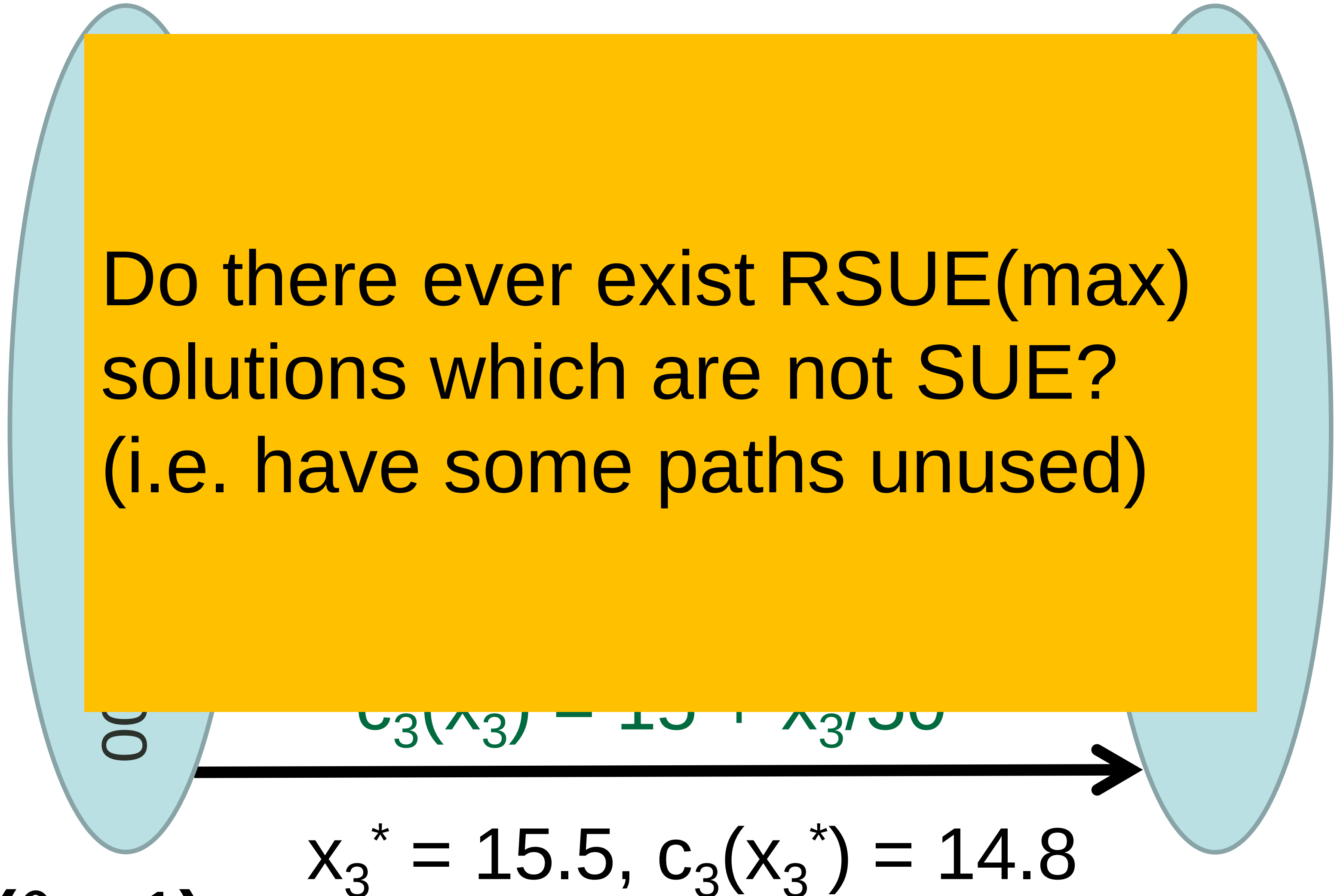
Only RSUE(max) solution (= SUE)



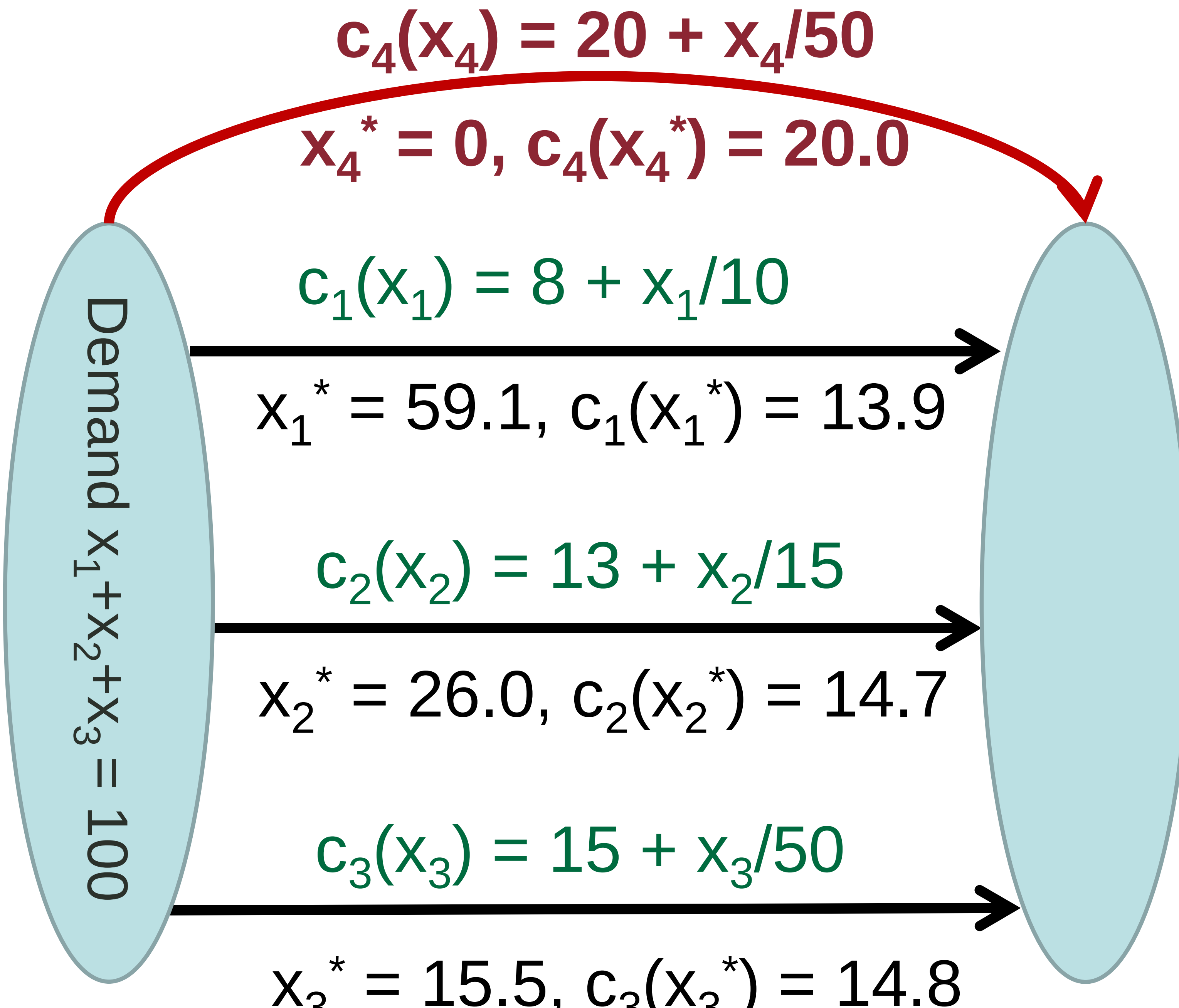
(θ = 1)

Only RSUE(max) solution (= SUE)

Do there ever exist RSUE(max) solutions which are not SUE?
(i.e. have some paths unused)



(θ = 1)


$c_4(x_4) = 20 + x_4/50$
$x_4^* = 0, c_4(x_4^*) = 20.0$
$c_1(x_1) = 8 + x_1/10$
$x_1^* = 59.1, c_1(x_1^*) = 13.9$
$c_2(x_2) = 13 + x_2/15$
$x_2^* = 26.0, c_2(x_2^*) = 14.7$
$c_3(x_3) = 15 + x_3/50$
$x_3^* = 15.5, c_3(x_3^*) = 14.8$
Demand $x_1+x_2+x_3 = 100$

($\theta = 1$)

$c_4(x_4) = 20 + x_4/50$

This implies that this four-route problem has 3 RSUE(min) and 2 RSUE(max) solutions.

Good to have so many solutions?

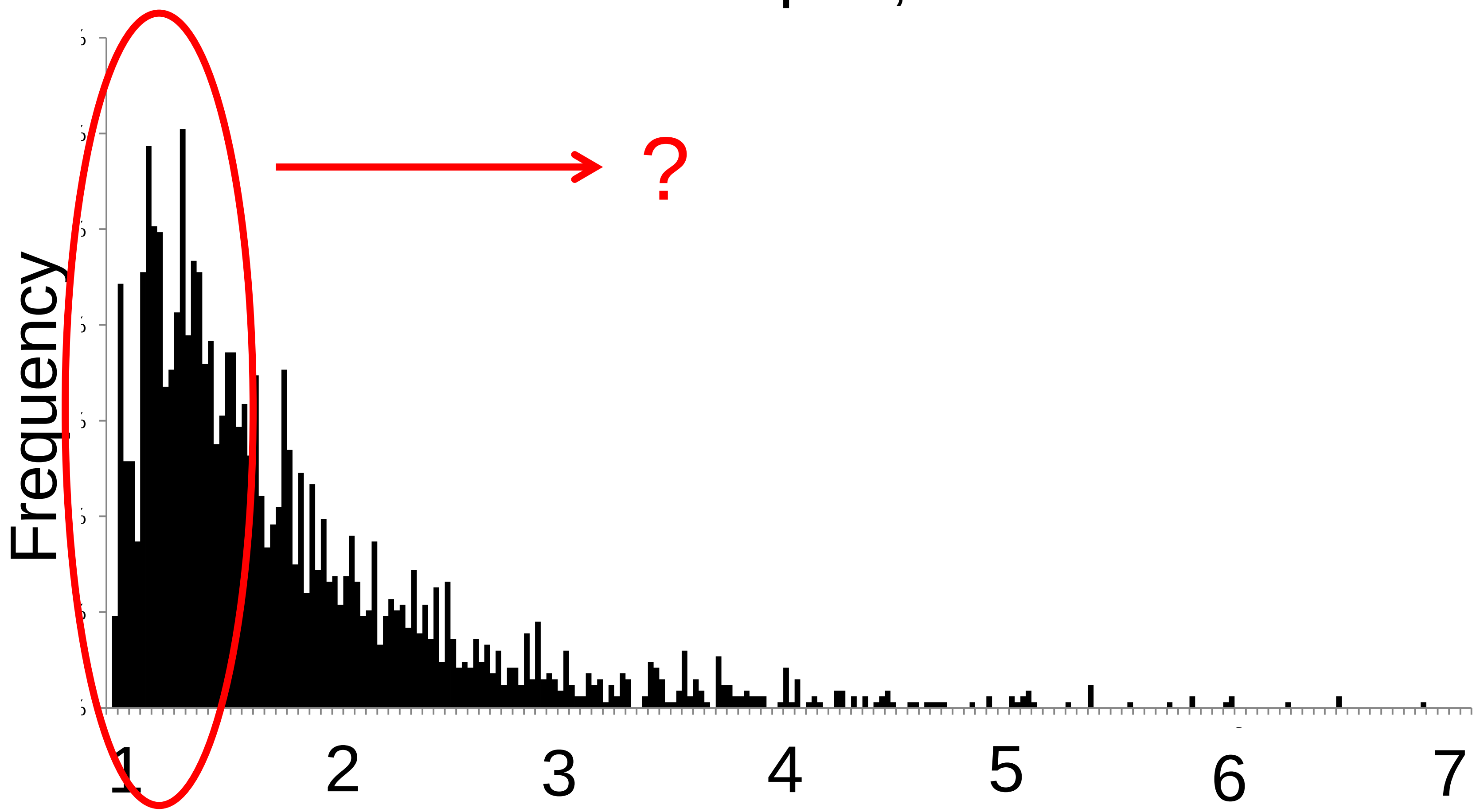
Could we control the number of possibilities in any way?

$x_3^{\wedge} = 15.5$, $c_3(x_3^{\wedge}) = 14.8$

($\theta = 1$)

DUE model in a larger network: unused paths

Relative costs unused paths, Sioux Falls



Horizontal axis = Cost of an unused path / Cost of any used path for all ODs

$$\text{Horizontal axis} = \frac{\text{Cost of an unused path}}{\text{Cost of any used path}} \quad \text{for all ODs}$$

Reminder: RSUE(min) model

for each OD movement …

1. Used routes used in proportion to $\exp(-\theta c_r)$.

2. Reference Cost 1 = min {cost of a used path}

3. Unused path cost $\geq$ Reference Cost 1.

RSUE with Threshold: RSUET(min , τ min)

for each OD movement …

1. Used routes used in proportion to $\exp(-\theta c_r)$.

2. Reference Cost 1 = min {cost of a used path}

3. Unused path cost $\geq$ Reference Cost 1.

4. Reference Cost 2 = τ min {cost of a used path}

5. Used path cost $\leq$ Reference Cost 2. ($\tau \geq 1$)

RSUET(min, τ min) model ($\tau \geq 1$)

Any RSUET(min, τ min) is also a RSUE(min) solution, but not *vice versa*.

For example, if:

- we choose a threshold of τ =1.2 (i.e. any used route must be no more than 20% more costly than the best used route)
- we have an RSUE(min) solution with three routes used, with costs 10, 11, 13
- this is not a RSUET(min,1.2min) solution because 13 is more than 1.2 $\times$ 10.

General definition: RSUET(Φ, Ω) model

- Given operators: Φ (e.g. min, max)
  Ω (e.g. τ min, α + min) ...

A route flow $\mathbf{x} \in G$ is an RSUET(Φ, Ω) iff $\forall\ r \in R_m$ and $\forall\ m = 1,2,\ldots,M$:

$$x_{mr} = 0 \Rightarrow r \notin \tilde{R}_m \text{ and } c_{mr}(\mathbf{x}) \geq \Phi(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\})$$

$$x_{mr} > 0 \Rightarrow r \in \tilde{R}_m \text{ and } x_{mr} = d_m P_{mr}(\mathbf{c}(\mathbf{x}) \mid \tilde{R}_m)$$

$$\text{and } c_{mr}(\mathbf{x}) \leq \Omega(\{c_{ms}(\mathbf{x}) : s \in \tilde{R}_m\})$$

Conclusions and Future Research

- It is possible to write self-consistent conditions for a family of equilibrium models which:

(a) disperse traffic among more than minimum cost routes (limitation of DUE, using SUE)

(b) avoid the SUE problem of dispersing to all possible routes, using DUE analogy to distinguish used and unused paths.

- Algorithms are needed to implement these methods in large real-life networks.
- Validation of the equilibrated choice sets should be performed.

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References

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